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Foam flow phenomena in sudden expansions and contractions

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Abstract

A sudden change in flow area induces complex transient processes in foams, which increase in significance with increasing flowrate. The effects are a function of the method of foam generation and, hence, depend on the initial foam structure. In vertical upflow at a sudden expansion or contraction, a substantial fraction of the liquid in the foam drains back upstream. This leads to a greatly reduced liquid holdup downstream and a recirculating flow regime upstream. For the area ratios of the fittings investigated, the extent of liquid holdup reduction was less pronounced in the case of a sudden contraction. At low gas rates, a plug flow regime usually existed on either side of the fitting; however, with an increase in gas throughput the presence of the fitting led to the establishment of a recirculating flow regime upstream. At sufficiently high flowrates complete breakdown of foam structure occurred resulting in the onset of a 'flooding' regime downstream. Foam rheology was successfully described by a two-parameter power-law model, and the friction factor for foam flow could be described by a simple explicit relationship. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Foam flow; Contraction; Expansion; Friction factor; Gamma-ray absorption; Liquid holdup; Pressure drop

1. Introduction

Foaming is a widespread phenomenon across a variety of industries. It is exploited in such processes as displacement of oil from porous strata in enhanced oil recovery, and in the use of foam as a drilling fluid and sand cleanout fluid. Foaming of gas–oil mixtures is also being considered as an alternative to slug-catchers on oil pipelines, especially in hilly terrains which favour the formation of large gas slugs that can seriously affect process plant operability. Foam is used as

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a pipe conveyor of pulverised coal, as a disperser of pigments in paper coating and textile finishing, as a fire-fighting agent, and in foods, e.g., ice cream, mousses, whipped cream. Foaming is also used in the separation and purification of surfactants, metallic ions, dyes, proteins, enzymes and other surface-active biological components. In all cases, the presence and nature of the foam can determine both the economic and technical successes of the industrial process concerned.

A gas–liquid foam generally forms when a gas is intimately mixed with a liquid containing one or more surface-active agents. It represents a structured two-phase fluid in which gas bubbles are separated by a network of interconnecting thin liquid films; the volume fraction of the continuous liquid phase is usually small in dry foams but can be substantial in wet foams. Foam flows are challenging because foams have a complex structure with striking metastability characteristics. It is not surprising, therefore, that though applications involving the flow of foam are numerous, there is still a lack of information on foam flow behaviour through pipes and fittings. In particular, the behaviour of foams flowing through pipes and fittings with diameters greater than a few millimeters has received insufficient attention. In one of the rare reviews in the last 20 years, Assar and Burley (1984) identified a severe lack of information on quantitative rheology and flow behaviour of foams. Since then, only modest progress has been achieved and many issues remain unaddressed or unresolved in this area. The literature on foam flow in pipes has been recently reviewed by Deshpande and Barigou (2000).

Pipe fittings are essential components of any flow system and can have an important influence on the flow. Thus, foam flow rigs invariably contain a variety of pipe fittings which may involve a change in flow area (e.g., enlargement, contraction), in flow direction (e.g., bends), or a change in both flow area and direction (e.g., valves). Whilst, the flow of foams in straight pipes has been the subject of a number of investigations (see, e.g., Hoffer and Rubin, 1969; Calvert and Nezhati, 1986; Calvert, 1990; Enzendorfer et al., 1995; Hanselmann and Windhab, 1996; Deshpande and Barigou, 2000), flow through pipe fittings has hardly been studied and there is very little information available on the effects of flow constrictions on foam flow characteristics.

For a Newtonian fluid the energy loss through a pipe fitting arises from internal friction and flow separation, and is approximately proportional to the square of the flowrate. For foams the situation is much more complex, and pressure drop through a geometric constriction cannot be easily predicted. Besides pressure drop and flow patterns, pipe fittings can also affect foam structure by affecting liquid hold up, bubble size and foam homogeneity. For example, Calvert (1988) who seems to be the only investigator to have looked at foam flow through fittings, observed that the foam downstream of a globe valve was unstable, of poor non-homogeneous quality, and almost degraded to a slug flow of alternate large bubbles and very wet foam. It is common knowledge in the fire-fighting industry that a globe valve can degrade a foam completely to an air/water slug flow, but a gate valve at the same pressure drop would give a stable homogeneous foam texture (Calvert, 1988). Foam structure, homogeneity, and liquid holdup are crucial in most applications, and it is necessary to understand the effect of fittings on these parameters.

In this paper, we report on an experimental study of foams flowing through a sudden expansion and a sudden contraction. The phenomena introduced by a sudden change in the area of flow has been characterised in terms of the effects on liquid holdup, pressure drop, and bubble size. The interaction of the fittings with foams having various initial structures produced by different foam generators has been investigated. The pressure drop characteristics of the foam flows upstream and downstream of the fittings have been analysed in terms of the fanning friction factor.

2. Experimental

The experimental set-up is shown in Fig. 1. The vertical foam column consisted of two glass pipes of 800 mm length connected by bolted flanges. The lower section was of 27 mm diameter. In the case of the sudden expansion, the upper section consisted of a glass pipe of 38 mm diameter, whereas a glass pipe of 15.5 mm diameter was used for the sudden contraction, thus giving a ratio of downstream to upstream cross-sectional areas, A_D/A_U, of 2.0 and 0.33 for the sudden expansion and contraction, respectively.

The surfactant used to stabilise the foam was sodium dodecyl benzene sulphonate (NaDBS) in distilled water at a concentration of 0.4 kg m^{-3}, which is equivalent to its critical micelle concentration. The viscosity of the surfactant solution was 1.0 mPa s and the surface tension was 37.3 mN m^{-1}. The foam was generated pneumatically by sparging prehumidified nitrogen gas through a pool of surfactant solution at the base of the column. The gas spargers used were a single orifice of 1.5 mm diameter, and two sintered glass discs of 40–100 μm and 100–160 μm porosity.

The foam carried over was collected in a large reservoir containing foamable liquid, where it was destroyed by a mechanical foam breaker and the foamate returned to the column bottom by

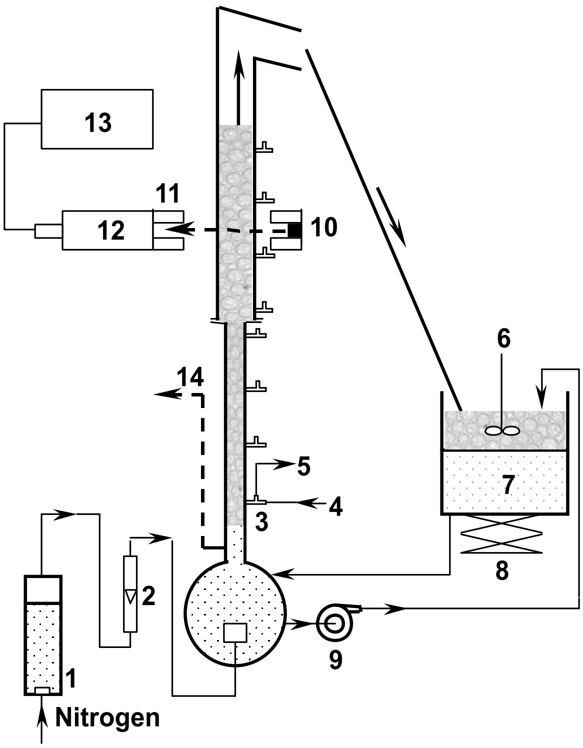

Fig. 1. Experimental set-up. 1: humidifier; 2: rotameter; 3: pressure tapping; 4: gas side bleed; 5: connection to micromanometer; 6: foam breaker; 7: reservoir of surfactant solution; 8: level adjustment; 9: pump for recirculation; 10: gamma-ray source Am^{241}; 11: collimator; 12: scintillator and photomultiplier; 13: Counter; 14: liquid sample to UV spectrophotometer.

gravity. The foam–liquid interface in the column was maintained at a constant level throughout the experiments by adjusting the level of the reservoir. In order to compensate for surfactant fractionation in the column, foamable liquid was continuously circulated from the large reservoir using a peristaltic pump. To check that this mechanism provided a liquid supply for foam generation with a consistent surfactant concentration, the liquid at the bottom of the column was sampled under all experimental conditions, and its NaDBS concentration measured on a UV spectrophotometer at a wavelength of 225 nm. The surfactant concentration of the liquid pool in the column was always well within ±10% of that in the reservoir. Thus, it could be safely assumed that there was no significant surfactant fractionation or evaporation effects on the foams produced.

The mass flowrate of foam was measured by collecting and weighing foam samples at exit. Steady-state operation was always allowed to establish before taking measurements. Steady state was indicated by maximum pressure readings and a steady rate of foam flow at the column exit. At high flowrates, this only took about 10 min, but at low flowrates it took a few hours; for a slow rising foam, a long time is required to achieve a time-independent liquid holdup distribution and a lubricating liquid layer of constant thickness on the pipe wall.

The piezometric pressure drop was measured across the fitting and along the straight pipe sections either side of the fitting, at four positions 20 cm apart over a length of 60 cm. A sensitive strain-gauge type micromanometer (Furness Controls, model FCO14-2) capable of detecting gauge pressure heads down to 0.1 mm of water was used for this purpose. The pressure tappings were each supplied with a side nitrogen bleed through a 0.5 mm diameter capillary to prevent foam from flowing inside the tappings, without affecting pressure readings. The flow of the gas bleed was just sufficient to keep the tapping clear from foam, and all the pressure in the gas was lost through the capillary used. This was checked by measuring pressure with and without the gas bleed when the tapping was clear of foam and no difference was detected.

The local section-average liquid holdup in the foam, one of the main parameters determining foam quality and structure, was measured using a gamma-ray absorption technique. The effects of gas flowrate on foam liquid holdup were studied by taking measurements at positions 20 cm upstream and downstream of the fittings.

Gamma-ray absorption has been widely used in two-phase pipe flow but it has rarely been used in foams. Radiation energy associated with a gamma-ray source is attenuated to a much greater degree in liquid than in gas, and the level of absorption in a gas–liquid mixture such as a foam depends on the relative volume fractions of the two phases (Hewitt, 1978, 1982).

In applying the method, a radioactive source Am^{241} of 11.38 μCi strength, and a radiation detector were mounted diametrically on either side of the foam column on a vertical travelling mechanism. Thus, measurements could be made at any location along the column, in particular at the inlet and outlet of the fitting. The local liquid holdup, ε_L, was obtained by measuring the intensity I_{GL} transmitted through the foam, and using the well-known relationship (Hewitt, 1978, 1982):

$$\varepsilon_L = 1 - \frac{\ln(I_{GL}/I_L)}{\ln(I_G/I_L)}, \tag{1}$$

where I_G is the intensity of the gamma-ray beam transmitted through the column containing air only, and I_L is the intensity of the beam transmitted through the column containing foamable liquid only. The radioactive source was selected to give enough radiation strength and high sensitivity.

The gamma-ray technique has the advantage of being non-intrusive and accurate. An analysis of the method's accuracy and precision in bubbly flows can be found in Hewitt (1982). Other methods used in the literature to determine local liquid holdup, which involve the withdrawal and analysis of foam samples, were found to disrupt the flow and give erroneous measurements. In the absence of an independent technique suitable for measuring local gas/liquid holdup in foams, the accuracy and precision of the gamma-ray densiometer was tested by measuring gas holdup in non-foaming bubble columns of various diameters. A radiation count period of 3 min was adopted though 1 min was usually sufficient for a satisfactory count. The holdup values were within $\pm 3\%$ of those obtained by measuring the gas–liquid dispersion level in the bubble columns, and the precision of the technique was better than $\pm 0.5\%$, in tubes of diameter larger than 15 mm. The accuracy of the method decreased for narrower tubes.

Bubble size distribution was determined by photographing the foam in flow through the transparent column wall using a digital camera (Ricoh RDC 300). The digital images were analysed on Q-Win-Pro software supplied by Leica Imaging Systems. The sample size was about 200 bubbles for foams with fine structures generated by sintered discs, and about 40 bubbles for coarse foams generated by the single orifice. The parameters of the bubble size distribution such as the number-mean bubble diameter, d_{10}, were reproducible to better than 1%. The effects of gas flowrate on foam bubble size were studied by taking measurements at positions 20 cm upstream and downstream of the fittings. Bubble compressibility effects along the column were negligible as the operating pressures were close to atmospheric.

3. Results and discussion

3.1. Sudden expansion

The experimental results indicated that a sudden change in the area of flow, i.e., a sudden contraction or expansion, introduces complex changes in foam structure which increase in significance as the flowrate increases. Foam structure is mainly determined by its bubble size and liquid holdup which are dictated to a great extent by the method of gas injection employed. A fine porosity sparger generates a wet foam with small spherical bubbles separated by relatively thick liquid films. This was confirmed by measuring the liquid holdup in the foam generated by the different spargers used, i.e., the single orifice (1.5 mm dia.) and the sintered glass discs (40–100 μm; 100–160 μm). The sintered discs produced wet foams with an average bubble diameter of the order of 1 mm or less, whilst the single orifice generated a drier polyhedral foam with an average bubble size of about 5 mm.

For a foam flowing in steady state in a vertical pipe of constant cross-section, gamma-ray measurements showed that the liquid holdup profile is uniform away from changes in cross-section and flow direction. An equilibrium is established between the processes of liquid entrainment by the rising foam and continuous foam drainage which is controlled by the interstitial liquid flow within the network of liquid films. The foam films drain into the adjoining plateau borders due to capillary suction, which in turn, form a network through which the liquid is driven downwards by gravity. When a wet foam such as that generated by the sintered sparger of porosity 100–160 μm flowed through the sudden expansion, the dynamic interaction with the fitting resulted in interstitial liquid being released from the foam, thus, resulting in a drier foam downstream. This liquid drained back through the rising foam and, as a consequence, the liquid holdup upstream increased substantially throughout the column.

Fig. 2 depicts a typical liquid holdup profile along the column, with much higher liquid fractions upstream. Downstream of the expansion, there is little variation in liquid holdup with column height where the foam is under a quasi-equilibrium regime. Upstream of the expansion, the liquid holdup profile is again nearly flat except in the vicinity of the fitting where it rises somewhat.

Fig. 3 shows the variation of liquid holdup at positions 20 cm upstream and downstream of the fitting as a function of gas flowrate. The effects of the sudden enlargement in flow area on liquid holdup distribution increased in significance with flowrate, leading to very high liquid fractions upstream. The increased liquid downflow resulted in intense recirculation patterns and, hence, led to the establishment of a recirculating flow regime upstream of the expansion even at the lowest flowrates. Onset of bubble flow was observed in the lower part of the column at the highest liquid holdups.

Increases in gas flowrate generated liquid eddies in the expansion. These increased in size until at a gas flowrate of about 1.17×10^{-5} m^3 s^{-1}, the foam structure broke down completely to give rise to a pool of liquid inside the expansion. Gas bubbles rose through the liquid pool and fresh

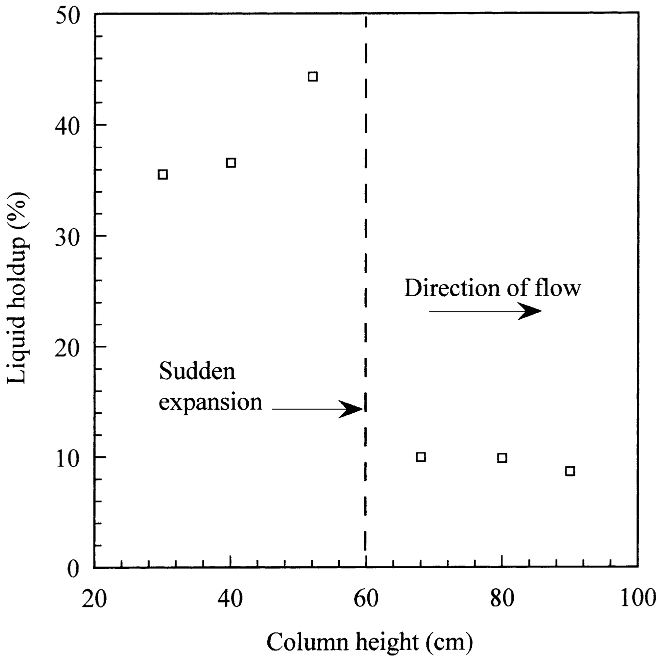

Fig. 2. Liquid holdup profile in foam flow through sudden expansion: 100–160 μm porosity sparger; gas flowrate = 6.67×10^{-6} m^3 s^{-1}.

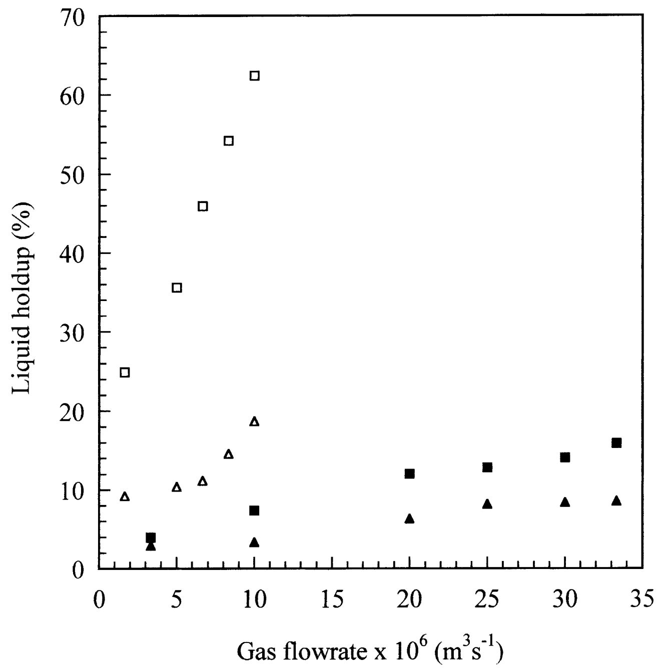

Fig. 3. Liquid holdup upstream and downstream of sudden expansion: 100–160 μm porosity sparger – upstream (□), downstream (△); 1.5 mm single orifice sparger – upstream (■), downstream (▲).

foam was regenerated at the liquid surface. The liquid pool which was initially a few millimetres deep, increased in depth and eventually the whole section of the column downstream of the expansion was flooded with liquid. The existence of this liquid pool also led to more intense recirculation patterns upstream. The onset of this 'flooding' regime is illustrated photographically in Fig. 4. Calvert (1988) too reported observations of foam breakdown resulting from flow through a globe valve. Consequently, when designing a flow system for foam transport, it is important to consider the possible effects of including such fittings on the end-of-pipe structure of the foam.

The flowrate at which disintegration of foam takes place is a function of its initial structure and bubble size. Thus, flooding occurred at the low flowrate of 0.33×10^{-5} m^3 s^{-1} when a very fine sintered glass sparger of porosity 40–100 μm was used, compared to 1.17×10^{-5} m^3 s^{-1} when the 100–160 μm sparger was used. The range of experiments with the very fine sparger was, therefore, very limited. Overall, the effects of the sudden expansion on foam structure were less significant in the case of the coarse polyhedral foam generated by the single orifice. No flooding of the expansion was observed over the range of experimental conditions used, though the liquid holdup increased upstream of the expansion. The effect of foam generator type on the liquid holdup is illustrated in Fig. 3. The single orifice gave rise to much drier foam compared with the sintered discs, and the expansion effects are much less significant.

Consistent with previous findings, the piezometric pressure varied linearly along the straight pipe sections (Deshpande and Barigou, 2000). The piezometric pressure gradients, obtained from the four pressure measurements along each straight pipe section upstream and downstream of the

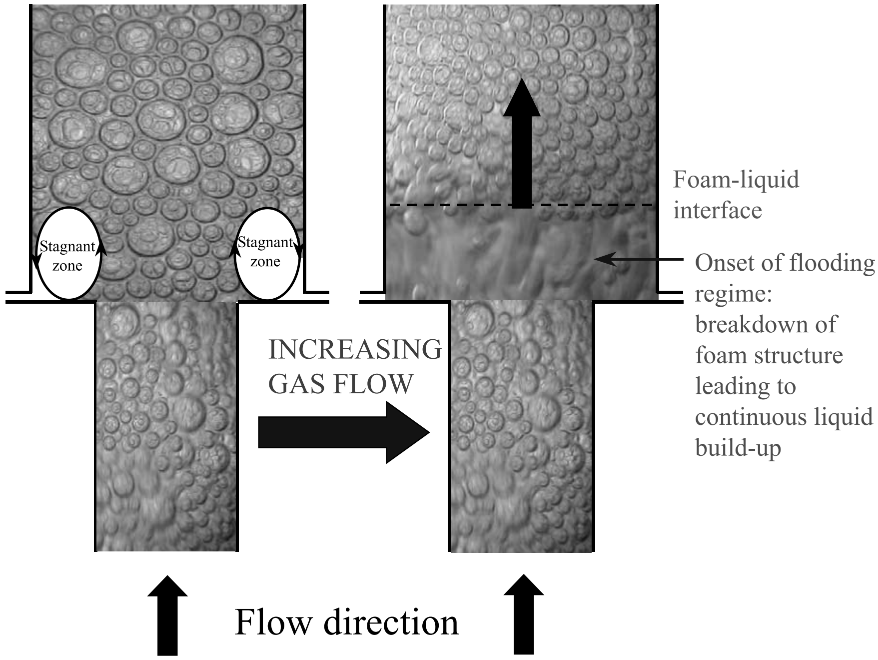

Fig. 4. Onset of flooding regime downstream of a sudden expansion.

expansion before the onset of flooding, are plotted in Fig. 5. For a given gas flowrate, the pressure gradient downstream of the expansion is much less than upstream. This reduction in pressure gradient is partly caused by the increase in area of flow and the associated reduction in liquid holdup, but is also probably a consequence of the changes in foam structure and flow regime. The local pressure drop across the sudden expansion itself was quite small, however, being equivalent to about one pipe diameter for all flow conditions studied.

Significant increases were observed in the number-mean bubble diameter, d_{10}, across the fitting; these were probably caused by a combination of factors including film breakage, bubble coalescence, and bubble expansion (see Fig. 6). Dry polyhedral foams as generated by the single orifice gas sparger incurred much less significant changes in bubble size. Such cellular foams with a cohesive structure where bubbles do not move relative to each other, exhibited good 'elastic' properties which allowed them to easily expand and contract and flow round sharp corners. Thus, they incurred much less significant structural damage compared to wet foams with a loose bubble structure.

3.2. Sudden contraction

Flow through a sudden contraction produced significant reductions in foam liquid holdup downstream, as shown in Fig. 7, though not as substantial as in the case of the sudden expansion. However, the mechanism involved is similar: when foam interacts with the contraction, flow separation occurs in the stagnant corner zones and adverse pressure gradients

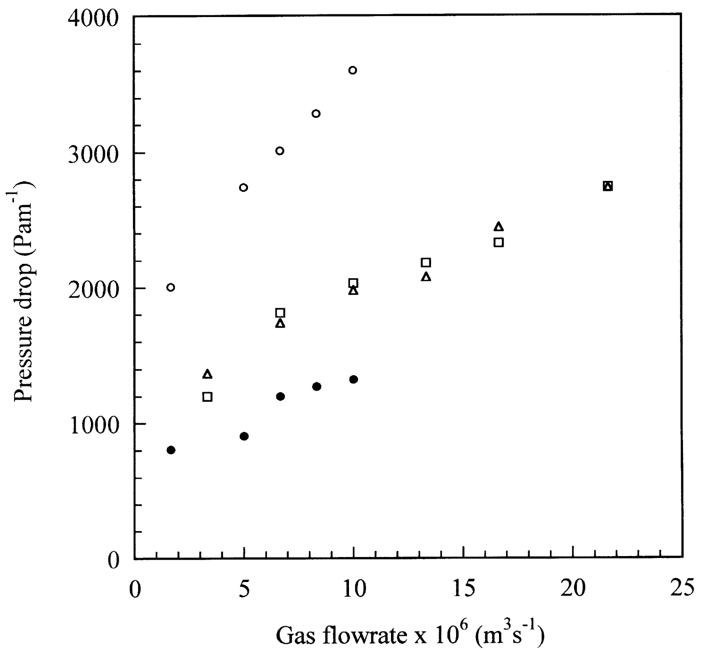

Fig. 5. Effect of sudden expansion and contraction on pressure drop: 100–160 μm porosity sparger: expansion – upstream (○), downstream (●); contraction – upstream (□), downstream (△)

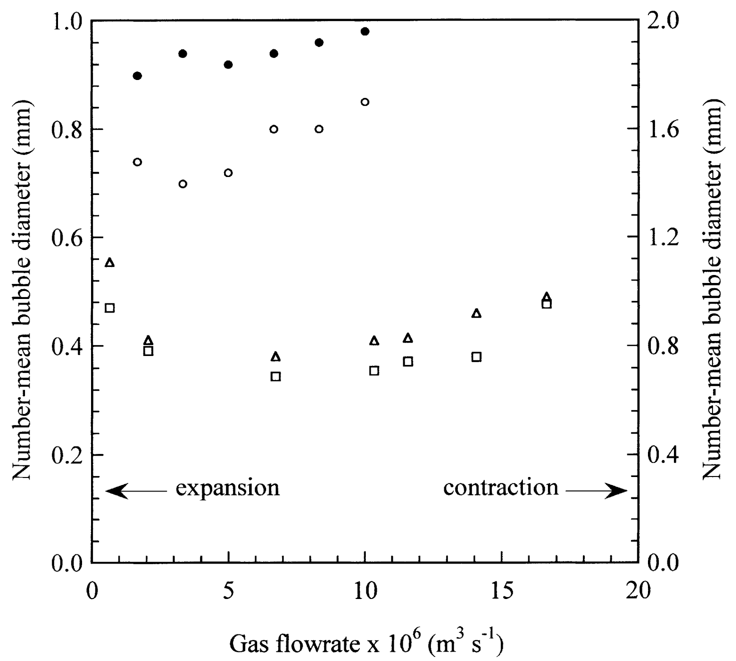

Fig. 6. Effect of sudden expansion and contraction on bubble size: 100–160 μm porosity sparger: expansion – upstream (○), downstream (●); contraction – upstream (□), downstream (△).

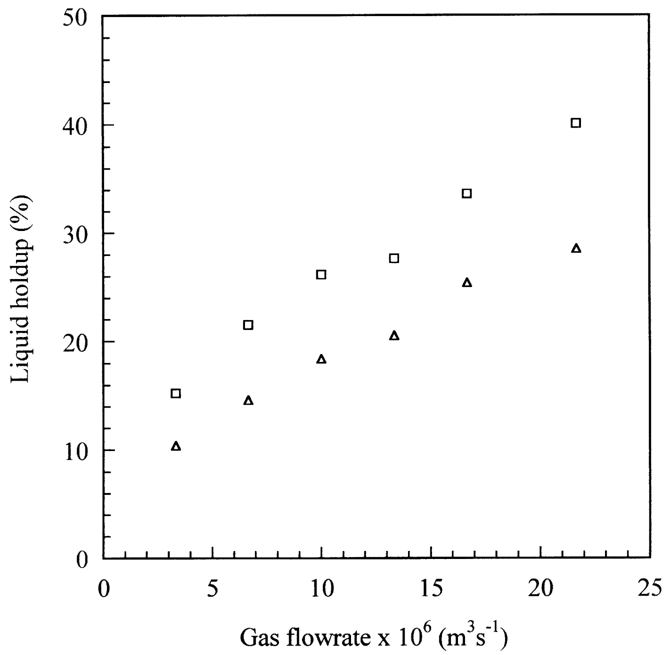

Fig. 7. Effect of sudden contraction on liquid holdup: 100–160 μm porosity sparger – upstream (□), downstream (△).

are set up which hinder the entrainment of the liquid in the foam. Although the increased liquid drainage led to the establishment of a recirculating flow regime upstream of the contraction even at the lowest flowrates used, the foam structure did not break down as in the case of the sudden expansion, and flooding was not observed. The pressure gradients, obtained from the four pressure measurements along each straight pipe section upstream and downstream of the contraction, are plotted in Fig. 5. At a given gas flowrate, the gradient downstream would in principle be expected to be much greater than upstream because of the reduction in area of flow. The fact that the values in Fig. 5 are similar indicates that, in real terms, the contraction caused a substantial reduction in pressure drop gradient in the pipe downstream. However, similar to the sudden expansion, the local pressure drop across the sudden contraction itself was not large and was only equivalent to about two pipe diameters for all flow conditions investigated.

The effect of the contraction on the number-mean bubble diameter is shown in Fig. 6. The average bubble diameter increased slightly downstream of the contraction. The trend in the variation of d_{10} with gas flowrate is probably a characteristic of the sintered glass sparger which consists of non-uniform orifices. A similar trend was previously observed by Hoffer and Rubin (1969). At low gas rates, the pressure drop across the foam generator is small and the gas flows mainly through the larger pores forming relatively large bubbles. As the gas flowrate increases, the pressure drop across the sparger increases and the smaller pores also become operational. Consequently, the fraction of small bubbles increases, thereby lowering the mean bubble diameter. With further increases in gas flowrate, bubbling at the sparger occurs under the constant frequency regime and d_{10} increases again.

3.3. Friction factor for foam flow

As discussed above, the pressure drop across a sudden expansion or contraction for the foams investigated is small and can be neglected compared to the overall pressure drop in the system. The foams flowing upstream and downstream of a sudden expansion or contraction are apparently different due to the important changes in liquid holdup and structure incurred across the sudden change in flow area. The effects on the foam hydrodynamics in the straight sections of the flow, and in particular the pressure drop characteristics are not immediately obvious, however.

By analogy with single-phase fluids, the pressure drop measurements were analysed in terms of the fanning friction factor, f, a very useful parameter for pressure drop calculations and hence for the design of flow systems. In order to derive friction factors for foams, the rheology of the foams flowing in the straight pipe sections upstream and downstream of the expansion or contraction was described by considering the gas–liquid system as a non-Newtonian pseudo-homogeneous fluid. Details of the analysis are given in Appendix A. The rheology of the foams that were subjected to the effects of a sudden enlargement or reduction in flow area was successfully represented by a power-law model (see Fig. 8), which is in accord with the results of Deshpande and Barigou (2000) in vertical straight pipes without flow constrictions.

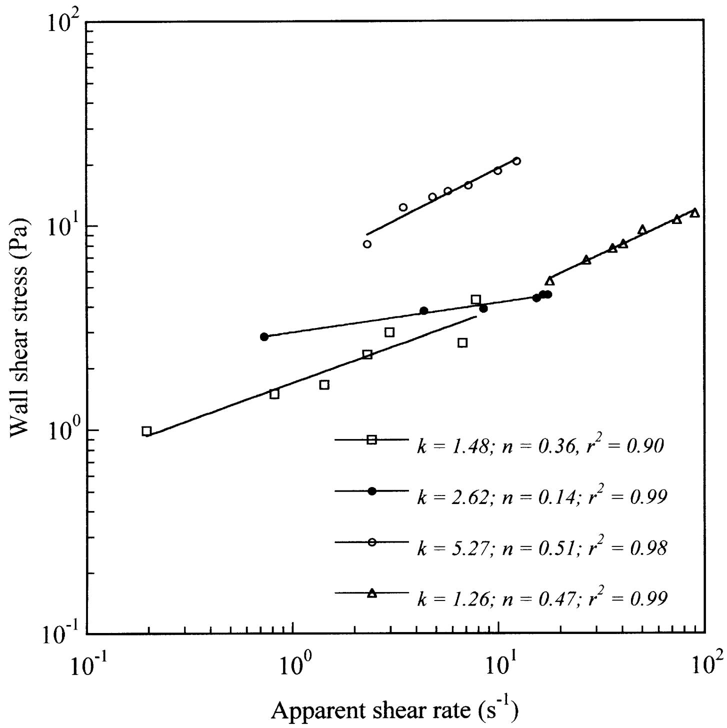

Fig. 8. Logarithmic plot of wall shear stress versus apparent wall shear rate for different foam structures upstream and downstream of sudden contraction: 100–160 μm porosity sparger – upstream (○), downstream (△); 1.5 mm single orifice sparger – upstream (□), downstream (●).

Friction factors are plotted in Fig. 9 as a function of the generalised Reynolds number, *Re*, for foam flowing in the straight pipe sections upstream and downstream of the expansion and contraction. The correlation

$$f = \frac{16.07}{Re^{0.99}} \quad (r^2 = 0.99) \tag{2}$$

provided a good fit for the data and is very close to the theoretical relationship $f = 16/Re$ expected for exact power-law behaviour (see Appendix A). This extends the earlier results of Deshpande and Barigou (2000) on pressure drop estimation in straight pipes to foam flow systems containing a sudden change in flow area despite the large variations in foam structure and flow regime. The improved agreement with the theory represented by Eq. (2) over the previous correlation ($f = 18.36/Re^{0.97}$) obtained by Deshpande and Barigou (2000) is due to the improved accuracy in local holdup measurement by gamma-ray absorption. In the earlier work, liquid holdup was determined by collecting and weighing foam samples at the pipe exit.

The results obtained here have practical significance in that pressure drop in foam flow through a series of pipes of different diameters can be calculated using a constant friction factor, $f = 16/Re$, along each length of constant cross-section, regardless of changes in foam structure and flow regime.

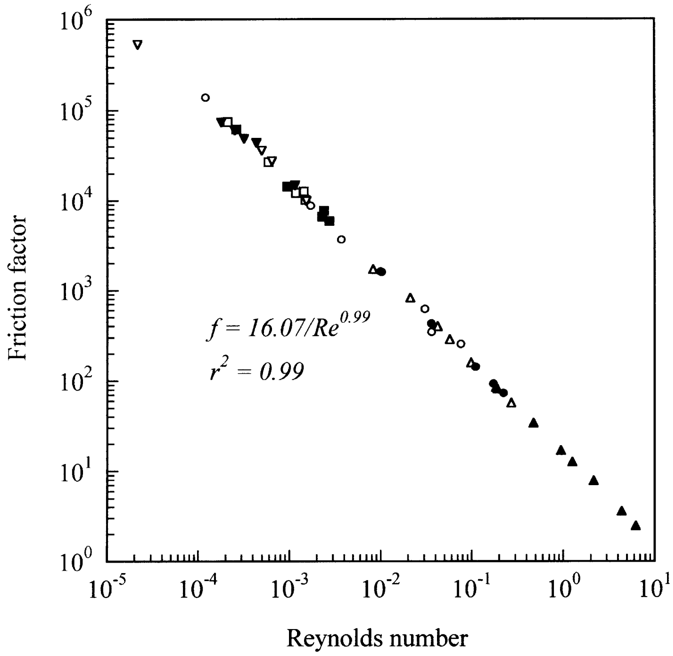

Fig. 9. Friction factors for foam flow in pipes of constant cross-section: 100–160 μm porosity sparger: expansion – upstream (□), downstream (■); contraction – upstream (△), downstream (▲); 1.5 mm single orifice sparger: expansion – upstream (▽), downstream (▼); contraction – upstream (○), downstream (●).

4. Conclusions

Sudden changes in the area of foam flow are characterised by complex phenomena. A sudden contraction or expansion influences the foam structure, liquid holdup, and the flow regimes upstream and downstream of the fitting. The liquid holdup decreases substantially downstream of the fitting, which results in intense recirculation flow patterns upstream and a dry foam downstream. A sudden expansion can eventually lead to complete breakdown of the foam structure with the onset of a flooding regime. These results show that the presence of fittings in foam flow systems can have serious effects on the end-of-pipe structure of the foam. These effects may have serious practical implications and have to be carefully considered when preservation of product structure is important. The pressure drop across a sudden expansion or contraction is small, equivalent to one or two pipe diameters. Foam rheology in the pipe sections on either side of a sudden expansion or contraction was successfully described by a power-law model. Pressure drop through a pipeline containing sudden changes in diameter can be calculated using a constant friction factor ($f = 16/Re$) along each length of pipe of constant cross-section, regardless of the variations in foam structure and flow regime.

Acknowledgements

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Appendix A. Friction factor analysis

Foam flowing in the straight pipe sections upstream and downstream of the expansion or contraction is considered as a non-Newtonian pseudo-homogeneous fluid. Calculations are, therefore, based on foam density and effective foam viscosity.

The rheology of the foams was described by a power-law model, thus

$$\tau_w = k\dot{\gamma}_w^n, \tag{A.1}$$

where τ_w and $\dot{\gamma}_w$ are the shear stress and shear rate at the pipe wall, respectively; and k and n are the flow consistency and behaviour indices, respectively.

The shear stress at the pipe wall is obtained from

$$\tau_w = \frac{D\Delta P}{4L}, \tag{A.2}$$

where D is the pipe diameter, L is the pipe length, and ΔP is the piezometric pressure drop along the pipe. The apparent wall shear rate, $\dot{\gamma}_a$, which corresponds to the maximum shear rate for a Newtonian fluid in pipe flow is given by

$$\dot{\gamma}_a = \frac{8u_F}{D} = \frac{32Q_F}{\pi D^3}, \tag{A.3}$$

where Q_F is the volumetric flowrate of foam, and u_F is the mean foam velocity in the pipe.

It has been suggested in the literature that $\dot{\gamma}_a$ be corrected for slip at the wall. In this work, no attempt was made to correct for wall slip as the aim was to establish a procedure for evaluating friction factors in foam flow to allow estimation of pressure drop, based solely on parameters that are readily measurable. In fact, in plug flow the corrected shear rate would be zero as foam flows entirely by slip over a lubricating thin liquid film at the wall, and it is the pressure drop associated with slip which is of interest in this case. Slip correction is discussed in more detail in Heller and Kuntamukkula (1987), Prud'homme and Khan (1996) and Deshpande and Barigou (2000).

The true wall shear rate, $\dot{\gamma}_w$, and $\dot{\gamma}_a$ are related through the well-known Rabinowitsch–Mooney relationship for the steady laminar flow of a time-independent fluid, thus

$$\dot{\gamma}_w = \left(\frac{3}{4} + \frac{1}{4}\frac{d(\ln\ \dot{\gamma}_a)}{d(\ln\ \tau_w)}\right)\dot{\gamma}_a \tag{A.4}$$

which is more commonly written for a power-law fluid as

$$\dot{\gamma}_w = \left(\frac{3n'+1}{4n'}\right)\dot{\gamma}_a, \tag{A.5}$$

where

$$n' = \frac{d(\ln\ \tau_w)}{d(\ln\ \dot{\gamma}_a)}, \tag{A.6}$$

i.e., n' is the slope of the $\ln(\tau_w)$ versus $\ln(\dot{\gamma}_a)$ curve. For a power-law fluid (Eq. (A.1)) it can be shown that $n' = n$.

The experimental data ($1.0 \leqslant \tau_w \leqslant 24$ Pa; $0.2 \leqslant \dot{\gamma}_a \leqslant 90$ s^{-1}) were plotted in the form of $\ln(\tau_w)$ versus $\ln(\dot{\gamma}_a)$ yielding the value of n as the slope. The value of the flow consistency k was then obtained from the intercept and the value of n. The k and n values varied with foam structure within the ranges $1.26 \leqslant k \leqslant 25.5$ Pa s^n and $0.14 \leqslant n \leqslant 0.6$ for the conditions investigated. Typical plots are shown in Fig. 8 for different foam flow conditions.

The fanning friction factor, f, for foam flow is defined as the ratio of the wall shear stress in the pipe to the kinetic energy per unit volume, i.e.,

$$f = \frac{\tau_w}{(1/2)\rho_F u_F^2}, \tag{A.7}$$

where ρ_F is the foam density determined from liquid holdup measurements at a position 20 cm either side of the fitting.

The generalised approach of Metzner and Reed (1955) relates flowrate and pressure drop for time-independent fluids in steady laminar flow, thus

$$\frac{D\Delta P}{4L} = k'\left(\frac{32Q_F}{\pi D^3}\right)^{n'} \quad \text{or} \quad \tau_w = k'\dot{\gamma}_a^{n'}. \tag{A.8}$$

This approach is based on the generalised Reynolds number, Re, defined as

$$Re = \frac{\rho_F u_F D}{\mu_e}, \tag{A.9}$$

where μ_e is the effective foam viscosity defined thus

$$\mu_e = \frac{\tau_w}{\dot{\gamma}_a} = k'\dot{\gamma}_a^{n'-1}. \tag{A.10}$$

For a power-law fluid, Eq. (A.1), it can be readily shown that $n = n'$ and $k' = k((3n+1)/4n)^n$. Hence, the effective foam viscosity is given by

$$\mu_e = k\left(\frac{3n+1}{4n}\right)^n \dot{\gamma}_a^{n-1} \tag{A.11}$$

and the Reynolds number in Eq. (A.9) then becomes

$$Re = \frac{\rho_F u_F^{2-n} D^n}{2^{n-3} k \left(3 + \frac{1}{n}\right)^n}. \tag{A.12}$$

It follows from the above analysis that for a foam obeying exact power-law behaviour, the friction factor as defined in Eq. (A.7) is given by

$$f = \frac{16}{Re}. \tag{A.13}$$

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